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Forecasting with Plant Process Models:

A Time Series Approach
II. Analysis

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FORECASTING WITH PLANT PROCESS MODELS: A TIME SERIES APPROACH. II. ANALYSIS, by Keith N. Crank, Statistical Research Division, Statistical Reporting Service, U. S. Department of Agriculture.

ABSTRACT

Nonlinear regression and time series models are used to forecast crop components and yields from three plant process models. The forecasts are not very good. The results show a dependence of the nonlinear models upon the particular plant process model employed. Since a large class of nonlinear models were used, we conclude that these plant process models do not provide enough information early in the growing season to forecast crop yields.

Keywords: plant process model, time series, nonlinear model, forecasting

CONTENTS

SUMMARY

The theory for the application of nonlinear regression and time series models for forecasting crop components and yields using outputs from plant process models (PPM's) was developed in a previous paper([1]). This paper presents an analysis of those models. Although the results are not impressive, one or more of these time series models could use data from a PPM to forecast crop yields.

If PPM's are used by SRS, the time series models presented here should be directly compared to other procedures for forecasting from PPM's. Therefore, future research on these models will depend entirely on Agency policies concerning the use of plant process models for yield forecasting.

In addition the results of this analysis indicate that these time series models are affected by the choice of PPM. If further research is to be done on these models, the research should be directed to a specific PPM. Any such PPM should be a final version which is considered suitable for Agency use.

INTRODUCTION

A previous paper ([1]) introduced a set of nonlinear regression and time series models which could be used to forecast crop yields from data generated by a plant process model (PPM). Five possible nonlinear models, two regression and three time series, were introduced and reasons were given for considering them. This paper analyzes those five models using data from three PPM's (CERES MAIZE, SORGF, and SOYGRO).

One reason for using these nonlinear models is to allow forecasts to be made using only current year data. This would prevent year to year changes in weather from influencing crop forecasts. Earlier papers ([3],[5],[6],[7],[8],[11],[12],[13],[18]) discuss the advantages of within year forecasting models.

The purpose of this paper is to present some analysis of the models described in [1]. The Background section briefly describes these five regression and time series models. A sixth model is introduced and a justification is given for dropping the two models which do not have a time series structure. The Analysis section describes the analysis which was done on the four remaining models, and presents the results of the analysis. The Conclusion states the usefulness of these time series models and mentions future work which may be beneficial. Three Appendices provide some detail which is not presented in the body of the paper.

BACKGROUND

Five models are to be compared to determine their ability to forecast crop yields. Each of the models uses data from a plant process model (PPM) to fit a regression (usually nonlinear). The parameters from these regressions are then used for forecasting end of season values for components of the plant. Some of these components can be used to forecast yield directly. Others must be used as input to another forecasting model. Although each of the models is described in [1], a brief description will also be given here.

<u>Logistic Growth Model</u>: The first model is the logistic growth model. This model has the functional form

$$W(t) = \frac{\alpha}{1+\beta\rho^{t}},$$

where $W(t)$ represents the weight of a plant part on day t, and α, β, and ρ are parameters to be estimated. The parameter α is of major interest since it represents the weight of the plant part at the end of the season.

Gompertz Model: The second model is the Gompertz model. This has the functional form

$$W(t) = \alpha\beta^{\rho^t},$$

where, as before, $W(t)$ represents the weight of the plant part on day t, and α, β, and ρ are model parameters. The parameter α has the same interpretation as before and is of primary interest. The parameters β and ρ may have different interpretations, but are not used explicitly in a forecast; hence their values do not present a problem.

Logistic Time Series: Each of these models can be written in terms of previous day's values (that is, as a time series). For the logistic model the functional form becomes

$$W(t) = \frac{\alpha W(t-1)}{(1-\rho)W(t-1)+\rho\alpha},$$

where $W(t-1)$ is the weight on the previous day and everything else is as before.

Gompertz Time Series: The Gompertz model, when written as a time series, has the form

$$W(t) = \alpha\left[\frac{W(t-1)}{\alpha}\right]^{\rho}.$$

In both of the time series representations the parameter β has disappeared. Thus only two parameters instead of three need to be estimated. Also, α still has the same interpretation as before.

Transformed Gompertz Time Series: The last model introduced in [1] is obtained by taking the log of both sides of the Gompertz Time Series model. The effect of taking logs is to produce a linear model instead of a nonlinear one.

$$L(t) = \beta_0+\beta_1 L(t-1),$$

where $L(t)$ and $L(t-1)$ represent the logs of $W(t)$ and $W(t-1)$ respectively, $\beta_0=(1-\rho)\text{Ln}(\alpha)$, and $\beta_1=\rho$.

When these models were fit to the data for an entire season, the variances of the residuals appeared to increase in time except for the transformed Gompertz time series model. For this model the variance of the residuals appeared to decrease with time. In linear regression these variances can often be made more uniform by a power transformation. This transformation was done for the Gompertz time series model using a power of 1/2, and improved the plots of the residuals against time. This sixth model is derived in Appendix A.

ANALYSIS

Data was available from three plant process models (PPM's). For the CERES MAIZE model documented in [4] there were ten different datasets each with four variables. For the soybean model SOYGRO described in [17] there were only two datasets each with two variables. SORGF, the sorghum model detailed in [10], had eight datasets with two variables.

All of the models except the square root transformation of the Gompertz time series were run for each of the variables for the entire season of data. For the nonlinear models this was done using PROC NLIN in SAS. The linear model was fit using PROC REG. Five output datasets were created, one for each model. Each of these output datasets was run through PROC ARIMA to test for autocorrelation of the residuals. The results are shown in table 1. The numbers in table 1 indicate the degree of an autoregressive model which would have to be fit to the residuals in order to eliminate autocorrelation. The abbreviation DNC indicates that the parameter estimates did not converge in PROC NLIN.

The results indicate that the logistic and the Gompertz models would be much worse than the other models either because they failed to converge or because the problem of autocorrelation is much worse than for the time series models. Thus these two models were dropped from further analysis. For the other models, since some autocorrelation was still present, an additional term was fit to the data to try to eliminate this problem. The details of the modified models are presented in Appendix B. Although the amount of autocorrelation was not the same for each model and each variable, the modified models only attempted to eliminate the first order autoregressive term. For the rest of this paper any references to a model will mean the modified form of that model.

The four remaining models were then compared in terms of their convergence properties. This was done by running the models using data only up to a certain point in time. This allowed us to determine which models could be used in early season forecasts, and also allowed us to find out which variables could be forecast. Thus for this part of the analysis, comparisons were made separately for each variable within a set of data for a given PPM.

Since the values for the grain component of the PPM's are zero until late in the season, it is not possible to use these models directly in estimating yield. However, these models can be used to estimate a maximum value for other plant components, and these estimated values can be used in another

Table 1a: The order of an autoregressive model which would have to be fit to the residuals to eliminate autocorrelation by dataset and model technique for CERES MAIZE.[1]

Variable	Dataset	Logistic Model	Logistic Time Series	Gompertz Model	Gompertz Time Series	Transformed Gompertz Time Series
leafwt	1	3	2	3	2	0
	2	2	1	DNC	1	4
	3	2	1	3	4	0
	4	4	3	4	3	3
	5	2	1	DNC	2	0
	6	2	2	3	2	4
	7	2	1	2	1	0
	8	5	5	DNC	4	2
	9	3	2	DNC	2	0
	10	4	3	4	3	2
stemwt	1	3	2	3	2	2
	2	2	1	2	1	1
	3	2	1	2	1	1
	4	4	3	4	3	4
	5	2	1	2	1	5
	6	2	2	2	2	1
	7	2	1	2	1	1
	8	3	2	3	2	4
	9	3	2	3	2	4
	10	2	1	2	1	3
earwt	1	DNC	1	DNC	2	0
	2	DNC	1	2	1	1
	3	DNC	3	3	2	2
	4	DNC	2	DNC	3	0
	5	DNC	3	2	1	0
	6	2	4	DNC	2	4
	7	2	2	DNC	2	0
	8	2	3	2	1	0
	9	2	0	DNC	3	0
	10	DNC	2	DNC	1	0
grainwt	1	DNC	1	DNC	1	1
	2	1	1	DNC	1	1
	3	DNC	1	DNC	1	1
	4	DNC	1	2	1	0
	5	DNC	1	DNC	1	1
	6	DNC	1	2	1	1
	7	DNC	1	2	1	1
	8	1	1	DNC	1	1
	9	DNC	1	DNC	1	1
	10	2	1	DNC	1	1

[1] DNC indicates that the parameter estimates did not converge using PROC NLIN.

Table 1b: The order of an autoregressive model which would have to be fit to the residuals to eliminate autocorrelation by dataset and model technique for SORGF.[1]

Variable	Dataset	Logistic Model	Logistic Time Series	Gompertz Model	Gompertz Time Series	Transformed Gompertz Time Series
grain	1	3	2	DNC	0	0
	2	2	2	DNC	0	0
	3	2	1	DNC	1	1
	4	DNC	1	DNC	0	0
	5	DNC	1	DNC	1	0
	6	DNC	1	DNC	1	1
	7	DNC	1	DNC	0	0
	8	2	1	DNC	1	1
total	1	3	2	4	1	3
	4	3	2	3	1	3
	3	2	1	3	1	3
	4	3	2	4	1	3
	5	5	4	5	1	2
	6	6	5	6	5	3
	7	5	4	5	2	3
	8	3	2	3	2	4

Table 1c: The order of an autoregressive model which would have to be fit to the residuals to eliminate autocorrelation by dataset and model technique for SOYGRO.[1]

Variable	Dataset	Logistic Model	Logistic Time Series	Gompertz Model	Gompertz Time Series	Transformed Gompertz Time Series
topwt	1	3	1	3	1	2
	2	3	2	3	1	1
seewt	1	3	1	DNC	1	1
	2	3	1	DNC	1	1

[1] DNC indicates that the parameter estimates did not converge using PROC NLIN.

type of model for forecasting yield. In our datasets regression models were created to forecast yield from the maximum value of these other components (stemwt for CERES MAIZE and total for SOYGRO). The forecasts made in this manner are identified in the tables and in the later discussion.

Since the same datasets are used both for estimating the parameters in the linear regression and for forecasting from those regression models, it could be argued that for the early dates this analysis is a test of goodness of fit rather than a test of the models. The author feels that this is not a problem for two reasons. First, this analysis is not designed to be a test of the model, only a preliminary study to see if the approach is feasible. Second, the independent variable which is used in the linear regression is a forecast rather than the actual value which was used to create the regression.

Tables two and three are only for the CERES MAIZE model and SORGF. With only two data sets the results for SOYGRO would be hard to interpret. However, for completeness, these results are presented in Appendix C.

The dates for CERES MAIZE indicate the last date for which data was used in making the forecast. For SORGF the headings denote the number of days from planting when the forecasts were made. Although it would have been preferable to treat the SORGF data in the same manner as the CERES MAIZE data, the actual dates were not available.

Table two shows the amount of autocorrelation left after the modified models were run. The numbers in table two are the number of samples for which the Q-statistic of Ljung and Box described in [9] is significant at an α level of .10 either up to lag 6 or up to lag 12. The Q statistic tests for lack of fit. If the statistic is significant the specified model may be inappropriate. In the CERES MAIZE model all of the August 1 and August 15 forecasts come from a regression of maximum stemwt on final grainwt. For SORGF all of the estimates at 70 days come from a regression of maximum total on final grain. Thus the entries for those columns are the same for both variables.

Table three shows the mean squared errors for three forecast dates and for the entire season. These are expressed as a percent of the true mean. The calculations were obtained as follows. Let

X_i = true value as obtained the PPM,

x_i = forecast of X_i, and

$\bar{X}$ = mean of X_i.

Then the mean squared error of the forecasts is

Table 2a: The number of samples from the CERES MAIZE model in which the Q-statistic of Ljung and Box was significant at an α level of .10 up to lag 6 or lag 12.

Variable	Model	August 1	August 15	September 1	Final
stemwt	Logistic Time Series	2	3	1	0
	Gompertz Time Series	2	3	0	0
	Transformed Gompertz Time Series	5	9	3	0
	Square Root Gompertz Time Series	0	1	2	2
grainwt	Logistic Time Series	2	3	0	0
	Gompertz Time Series	2	3	0	0
	Transformed Gompertz Time Series	5	9	0	0
	Square Root Gompertz Time Series	0	1	1	1

The data in the box comes from forecasting grainwt from the forecast of maximum stemwt using a linear regression model. The slope of the regression is .82 and the intercept is 3.35. Their standard errors are .17 and 14.4 respectively. The adjusted R-square for this model is .69.

Table 2b: The number of samples from the SORGF model in which the Q-statistic of Ljung and Box was significant at an α level of .10 up to lag 6 or lag 12.

Variable	Model	70 days	80 days	90 days	Final
total	Logistic Time Series	6	5	7	6
	Gompertz Time Series	6	5	5	4
	Transformed Gompertz Time Series	8	8	10	10
	Square Root Gompertz Time Series	6	6	9	9
grain	Logistic Time Series	6	2	3	0
	Gompertz Time Series	6	2	3	0
	Transformed Gompertz Time Series	8	5	4	0
	Square Root Gompertz Time Series	6	4	4	0

The data in the box comes from forecasting grain from the forecast of maximum total using a linear regression model. The slope of the regression is .25 and the intercept is 12.3. Their standard errors are .05 and 6.53 respectively. The adjusted R-square for this model is .69.

Table 3a: Root mean squared errors for CERES MAIZE (as a % of the mean). The numbers in parentheses are the mean and standard deviation of the ten samples used in the analysis.

Variable	Model	August 1	August 15	September 1	Final
stemwt (84.2,25)	Logistic Time Series	16	6	8	9
	Gompertz Time Series	55	15	9	9
	Transformed Gompertz Time Series	20	10	10	11
	Square Root Gompertz Time Series	25	15	7	7
grainwt (74.0,24)	Logistic Time Series	17	9	26	6
	Gompertz Time Series	49	13	39	18
	Transformed Gompertz Time Series	26	16	22	18
	Square Root Gompertz Time Series	26	14	33	19

The data in the box comes from forecasting grainwt from the forecast of maximum stemwt using a linear regression model. The slope of the regression is .82 and the intercept is 3.35. Their standard errors are .17 and 14.4 respectively. The adjusted R-square for this model is .69.

Table 3b: Root mean squared errors for SORGF (as a % of the mean). The numbers in parentheses are the standard deviation of the eight samples used in the analysis.

Variable	Model	70 days	80 days	90 days	Final
total (95.6,10)	Logistic Time Series	28	16	11	19
	Gompertz Time Series	16	12	21	41
	Transformed Gompertz Time Series	46	40	36	31
	Square Root Gompertz Time Series	13	12	15	23
grain (38.0,16)	Logistic Time Series	26	24	12	8
	Gompertz Time Series	17	16	8	12
	Transformed Gompertz Time Series	28	26	17	4
	Square Root Gompertz Time Series	17	18	8	6

The data in the box comes from forecasting grain from the forecast of maximum total using a linear regression model. The slope of the regression is .25 and the intercept is 12.3. Their standard errors are .05 and 6.53 respectively. The adjusted R-square for this model is .69.

$$MSE(x) = \frac{1}{n}\sum_{1}^{n}(x_i - X_i)^2.$$

The values in the table are $\frac{MSE(x)}{\bar{X}}$. Also shown in the table are the mean ($\bar{X}$), and the standard error of $\bar{X}$ divided by $\bar{X}$.

Comparing the four models to each other using tables 2 and 3, it appears that the transformed Gompertz time series model is the worst of the four. It has generally higher mean squared errors and a larger problem with autocorrelation than the other models. As was mentioned earlier it also has a problem with unequal residual variances. The Gompertz time series model is also a poor model. Although it appears to do well at times, it is not much better than at least one of the other models at those times. On the other hand when it does poorly, it is often much worse than the other models.

When the mean squared errors in table 3 are compared to the standard deviation of $\bar{X}$, the results are inconsistent. The time series models seem to do reasonably well for the SORGF variable grain 90 days after planting and for the CERES MAIZE variable stemwt on August 15 and on September 1. However, these models never do well for the SORGF variable total or for the CERES MAIZE variable grainwt. (The August 15 forecasts for grainwt actually come from the forecasts of stemwt.)

It should be noted here that the CERES MAIZE model does not use this type of model for grain fill. Instead this PPM uses a linear grain fill formula which is extrapolated at either end. This linear formula corresponds to the middle of the growth curve where the growth is approximately linear. The extrapolation is used to account for the small growth at either end of the curve. Our attempt to fit a nonlinear model to a linear function probably contributed to the poor results in forecasting grainwt in this model.

Conclusion

The results of this analysis are not impressive. However, these models do seem to be a viable method of forecasting if a plant process model is developed which provides an accurate description of plant growth under actual field conditions. Future research with these models is dependent on such a plant process model.

The results of the CERES MAIZE analysis indicate that the usefulness of any of these models for forecasting is extremely dependent on the structure of the plant process model. This means no more research on these models should be done unless it deals with a specific model which is to be tested for Agency use.

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APPENDIX A

In linear regression it is often useful to make a power transformation of the data to reduce the problem of heteroscedasticity of the residuals. If this is done to the Gompertz time series model(TSGOM), we get the following results:

$$W(t) = \alpha\left[\frac{W(t-1)}{\alpha}\right]^{\rho}$$

$$\left[W(t)\right]^{\lambda} = \left[\alpha\left[\frac{W(t-1)}{\alpha}\right]^{\rho}\right]^{\lambda}$$

$$= \alpha^{\lambda}\left[\frac{W(t-1)}{\alpha}\right]^{\rho\lambda}$$

$$= \alpha^{\lambda}\left[\frac{W(t-1)^{\lambda}}{\alpha^{\lambda}}\right]^{\rho}$$

Making the substitution

$$Y(t) = W(t)^{\lambda}$$

and letting $\alpha_1=\alpha^{\lambda}$, we obtain

$$Y(t) = \alpha_1\left[\frac{Y(t-1)}{\alpha_1}\right]^{\rho}$$

which is the same form as the original model. However, now our forecast is α^{λ} instead of α.

APPENDIX B

When autocorrelation exists in the residuals from a regression, the parameter estimates may not be consistent. This means that no matter how much data is available, the parameter estimates may not be close to their true values. Since the parameter estimates are used in forecasting, this is a major problem with these models. However, this problem can be corrected by fitting a different model to the data. The purpose of this Appendix is to show by an example how this is done, and to explain the effect of fitting a different model on the forecasts.

We will begin first with an example of how to correct the model to remove autocorrelation. Consider the Gompertz time series model

$$W(t) = \alpha\left[\frac{W(t-1)}{\alpha}\right]^{\rho} + e(t).$$

Suppose this model is fit to some data and the residuals show an autocorrelation structure. For the purposes of this example we will assume the residuals can be modeled as an AR(1) time series. Define

$$\tilde{W}(t) = \alpha\left[\frac{W(t-1)}{\alpha}\right]^{\rho}.$$

Let

$$e(t) = W(t)-\tilde{W}(t).$$

Then

$$e(t) = \gamma e(t-1) + a(t),$$

where γ is the autoregressive parameters from the time series, and $a(t)$ is a white noise process (that is, $a(t)$ is a process having mean zero and constant variance, and which is uncorrelated for distinct values of t). Then we can consider the new model

$$W(t) = \tilde{W}(t)+e(t)$$

$$= \tilde{W}(t)+\gamma e(t-1) + a(t)$$

$$= \alpha\left[\frac{W(t-1)}{\alpha}\right]^{\rho}+\gamma e(t-1) + a(t).$$

$$= \alpha\left[\frac{W(t-1)}{\alpha}\right]^{\rho}+\gamma\left[W(t-1)-\alpha\left[\frac{W(t-2)}{\alpha}\right]^{\rho}\right]+ a(t).$$

When we fit this model (estimating the parameters α, ρ, and γ), the residuals should be uncorrelated. Thus the parameter estimates will be consistent and will be useful for forecasting. However, since we have changed the model, we must ask what effect this has on our interpretation of the parameter α. The answer fortunately, is nothing. The parameter α still represents the limiting value of $W(t)$ for large t. This can be shown as follows. Let x be the limiting value of $W(t)$. Then

$$x = \lim_{t\to\infty} W(t)$$

$$= \lim_{t\to\infty}\left[\alpha\left[\frac{W(t-1)}{\alpha}\right]^{\rho}+\gamma(W(t-1)-\alpha\left[\frac{W(t-2)}{\alpha}\right]^{\rho})\right.$$

$$= \alpha\left[\frac{x}{\alpha}\right]^{\rho}+\gamma(x-\alpha\left[\frac{x}{\alpha}\right]^{\rho})$$

or

$$x-\alpha\left[\frac{x}{\alpha}\right]^{\rho} = \gamma(x-\alpha\left[\frac{x}{\alpha}\right]^{\rho});$$

This can only happen if either γ=1 or $\frac{x}{\alpha}=\left[\frac{x}{\alpha}\right]^{\rho}$. In the latter case we have $x=\alpha$ as desired. Unfortunately, the first case cannot be eliminated entirely. For an AR(1) time series the condition $\gamma=1$ corresponds to a process which is nonstationary (that is, differencing of the data is desired). If this is the situation, then any forecast for the limiting value of W(t) will be consistent with the data.

Note: As in the previous paper, we are assuming that W(t) has a limit for large t, and that a(t) becomes small for large t. As was mentioned in the previous paper, this violates the assumption of equality of variances, but we will assume that this is not a problem in the range of the data that we are using for forecasting.

APPENDIX C

These are the results from fitting the models to the data from SOYGRO. Since there were only two data sets, no conclusions can be made as to the ability of the models to forecast. This table is presented only for completeness.

Variable	Model	Actual	Oct. 1	Oct. 16	Nov. 1	Final
Topwt(78)	Logistic Time Series	984.56	938.12[1]	972.48	1019.97	1006.41
	Gompertz Time Series	984.56	1649.39[1]	1242.30[1]	1156.83	1067.16[1]
	Transformed Gompertz Time Series	984.56	1321.95	1296.11	1255.61[1]	1184.90[1]
	Square Root Gompertz Time Series	984.56	1745.67	1384.01	1253.26	1138.57[1]

Topwt(80)	Logistic Time Series	990.14	36.25[1]	156.51[1]	1032.08[1]	1012.06[1]
	Gompertz Time Series	990.14	1551.97	1236.23	1172.58[1]	1072.90[1]
	Transformed Gompertz Time Series	990.14	1583.45	1454.92	1368.21	1260.65[1]
	Square Root Gompertz Time Series	990.14	1658.98	1358.93	1252.63[1]	1135.94[1]
Seewt(78)	Logistic Time Series	427.32	NED[2]	256.17	106.27	436.25
	Gompertz Time Series	427.32	NED	382.24	494.12	457.03
	Transformed Gompertz Time Series	427.32	NED	301.75	431.44	445.78
	Square Root Gompertz Time Series	427.32	NED	326.15	471.80	456.85
Seewt(80)	Logistic Time Series	434.13	NED	93.60[1]	198.40	442.30
	Gompertz Time Series	434.13	NED	376.31[1]	543.75	461.22
	Transformed Gompertz Time Series	434.13	NED	338.78	455.91	456.62[1]
	Square Root Gompertz Time Series	434.13	NED	385.83[1]	533.42	468.82

[1] Box's Q-statistic is significant at an α-level of .10 either up to lag 6 or up to lag 12.

[2] There was not enough data to fit the regressions on this date for this variable.